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Fuel Minimal Takeoff Path of Jet Lift VTOL Aircraft

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Nomenclature

$[A]$	= matrix of aerodynamic/mass constant
$[B]$	= diagonal matrix of component, $b = T_m gc/(WV_s^2)$
C_{zx}, etc.	= aerodynamic coefficients; z and x are force and motion, respectively
g	= acceleration of gravity, m/s²
S	= main wing area, m²
T_x, T_z	= thrust x and z directions, respectively, kg
T_m	= maximum thrust
t	= time, s
W	= gross weight, kg
x,z	= coordinates of motion
ρ	= air density, kg s²/m⁴

Abstract

THE fuel minimal takeoff path analysis for jet-lift VTOL aircraft, both separate type and swivel type (Fig. 1), is presented. The analysis is aimed at providing an ideal value of the fuel consumption for VTOL aircraft design and operation.[1-3] The control law, derived from the Pontryagin's minimum principle,[4] results in a kind of dead-zone function which includes a singular solution. The paths are composed of two or three segments connected by the switching point. For the separate type, the singular part is necessary and unique; but for the swivel type, the singular part is not unique. Two methods of solution involving different handling of the singular part are considered.

Contents

The analytical systems are considered for the most simplifying assumptions, i.e., transition in a vertical flat plane, fixed attitude and aerodynamic coefficients, and neglect of weight change due to fuel consumption.

The equations of motion are:

$$(W/g)\ddot{x} = T_x - (1/2)\rho S(C_{xx}\dot{x}^2 - C_{xz}\dot{z}^2) \tag{1}$$

$$(W/g)\ddot{z} = T_z - W + (1/2)\rho S(C_{zx}\dot{x}^2 - C_{zz}\dot{z}^2) \tag{2}$$

The above equations are nondimensionalized using reference length c, reference velocity (horizontal target velocity) V_s, and nondimensional time $\tau = V_s t/c$.

For the separate type, the horizontal thrust is assumed constant, and the final reduced system is:

$$\dot{Z}_1 = Z_2 \tag{3}$$

$$\dot{Z}_2 = -c_1 Z_2 |Z_2| + c_2 \tanh^2(c_3\tau) - c_4 + u(\tau) \tag{4}$$

where Z_1 is the nondimensional altitude; Z_2 is the nondimensional vertical velocity; c_1, c_2, c_3 and c_4 are constant; and $u(\tau) = T_z/T_z(\max)$ is the specified control. The problem is to define the control u for the minimal fuel consumption cost

$$J = \int_0^{\tau_T} |u(\tau)|\, d\tau$$

(subscript T specifies target value) with the terminal conditions: $[Z_1, Z_2] = [0, 0]$ at $\tau = 0$, and $[Z_1, Z_2] = [Z_{1T}, Z_{2T}]$ at $\tau = \tau_T$. The problem is the "fixed time, fixed end point problem."

For the swivel type, the magnitude (norm) of the single thrust vector is constrained and the control is specified as $U = [u_1, u_2]'$, $\|U\| = \sqrt{u_1^2 + u_2^2} \leq 1$, and $u_1 = T_x/T_m$, $u_2 = T_z/T_m$. The final reduced system is

$$\dot{\bar{X}} = F[\bar{X};\tau] + [B]U(\tau) \tag{5}$$

where $\bar{X} = [X_1, X_2, X_3]'$, $F[\bar{X};\tau] = [A][X_1^2, X_2^2, X_2]' - [0, G, 0]'$, and $[A]$ and $[B]$ are constant matrices, $G = gc/V_s^2$, and states; X_1 is the nondimensional horizontal velocity, X_2 the nondimensional vertical velocity, and X_3 the nondimensional altitude. For the performance cost

$$J = \int_0^{\tau_T} |U(\tau)|\, d\tau,$$

the problem is to transfer the system from $\bar{X}(\tau = 0) = [0, 0, 0]'$ to $\bar{X}(\tau = \tau_T) = [1, X_{2T}, X_{3T}]'$ with minimum cost, and the problem is the "fixed end point, free time problem."

The control laws are derived from the Hamiltonian minimum conditions.

For the separate type, the co-states are p_1 and p_2, and the control laws are:

$$u(\tau) = 1 \qquad \text{if } p_2 < -1 \tag{6}$$

$$0 \leq u(\tau) \leq 1 \qquad \text{if } p_2 = -1 \tag{7}$$

$$u(\tau) = 0 \qquad \text{if } p_2 > -1 \tag{8}$$

Equation (6) defines the full thrust start path called "lift-off" and Eq. (8) defines the drifting final path called "coast." The singular path represented by Eq. (7) can be reduced to the nonsingular form and defines a constant vertical velocity intermediate path; $Z_2 = -p_1/(2c_1) = $ const. The constant value p_1 corresponds uniquely to the target altitude Z_{1T}; hence the solution is unique. Figure 2 shows examples of the Z_2 pattern ($\dot{z} = V_s G T_V Z_2$) for aircraft thrust/weight $T_V = T_z(\max)/W = 1.1$ and 1.05.

For the swivel type, with the co-state vector $P = [p_1, p_2, p_3]'$, the control laws are:

$$U = -U_m B'P/\|B'P\| \qquad \text{if } \|B'P\| > 1 \tag{9}$$

$$U = 0 \qquad \text{if } \|B'P\| < 1 \tag{10}$$

$$U = -C(\tau)U_m B'P/\|B'P\|, \quad 0 \leq C(\tau) \leq 1, \quad \text{if } \|B'P\| = 1 \tag{11}$$

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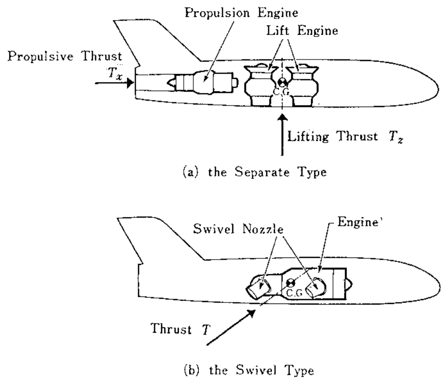

Fig. 1 Two basic types of jet-lift VTOL aircraft.

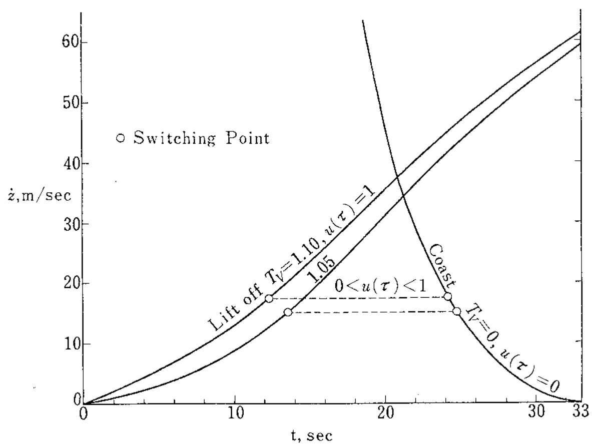

Fig. 2 Example of vertical velocity pattern for the separate type.

where U_m is the maximum scalar value of $\|U\|$.

Because the system is nonlinear and the necessary conditions are insufficient, numerical calculation algorithms for the nonsingular [Eq. (9)] and the singular [Eq. (11)] paths have been established to obtain numerical examples. Two groups of the nonsingular paths were found to exist: one starts from the $\bar{X}=0$ terminal called lift-off, the other is calculable for times less than the target time, and forms a group of coast paths. Generally both nonsingular paths are

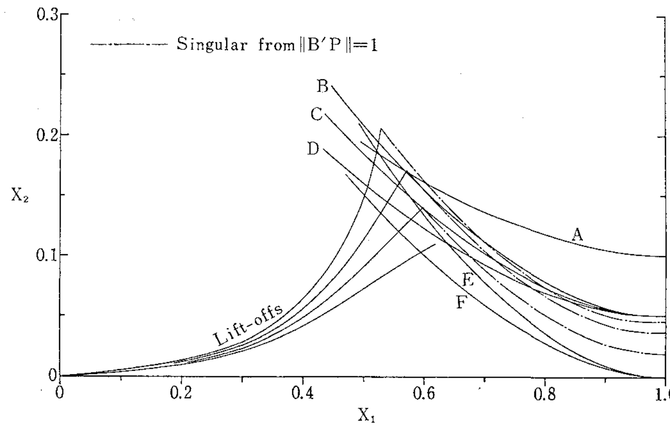

Fig. 3 Example of velocity vector pattern for the swivel type; lines A to F are a group of coast paths.

not continuous to the other terminal point, and terminate at the counter-moving $\|B'P\|=1$ boundaries; therefore one solution is a single switching from a liftoff path to a group of coast paths. Some checks indicate that optimality for the switching at the $\|B'P\|=1$ terminal occurs with higher X_{2T} and higher p_3 of the switched path.

While a usable singular path corresponding to Eq. (11) exists, it provides an unlikely solution to the intermediate path, and a different kind of solution using it after switching from the liftoff as the final path to the target was considered. An optimality check has shown that these singular solutions do not give the absolute minimum cost, and that the solution is not unique. Figure 3 shows an example of nondimensional velocity vector path combinations including singular paths.

References

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